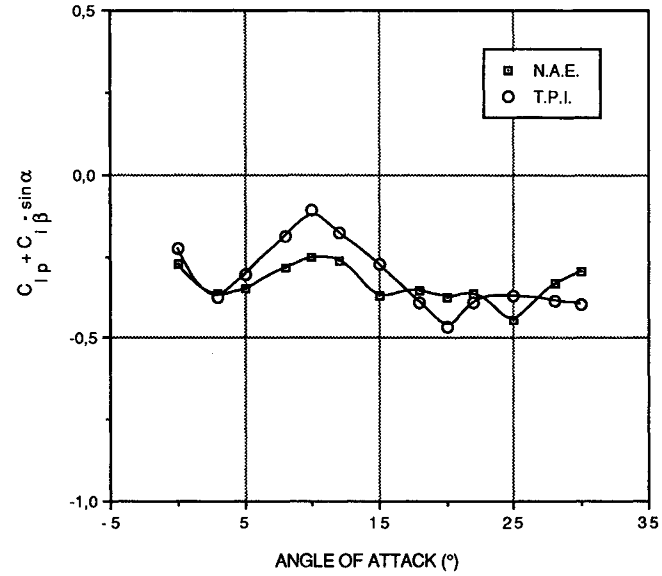

Fig. 4 Roll damping derivative.

Concluding Remarks

The results obtained with the calibration tests of the pitch/yaw and roll mechanism (related to the SDM) were in good accordance with the data published by NAE (IAR), DFVLR (DLR), and FFA, taking into account the different characteristics of the wind tunnels and of the model suspensions.

As a consequence, the research program has been extended to different model configurations, developed with the aim of investigating their aerodynamic behavior in a wide maneuvering field.

Acknowledgment

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Shape Sensitivities and Approximations of Modal Response of Laminated Skew Plates

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Introduction

THE derivatives of the natural frequencies and mode shapes of a generally laminated tapered skew plate with respect to various shape parameters are obtained. The frequencies and mode shapes of the composite cantilevered plate are initially determined as a function of a particular design variable using the Rayleigh-Ritz method. The derivatives of the eigenvalues and eigenvectors with respect to the shape variables are computed analytically and the results are compared with those obtained using the finite-difference method in order to confirm their accuracy. The four independent shape parameters considered are 1) the plate surface area, 2) the aspect ratio, 3) the taper ratio, and 4) the sweep angle. The eigenvalues and eigenvectors are then approximated over the range of the variable using linear, exponential, and pseudoexponential approximation schemes, and compared with the values obtained from reanalysis. Numerical results are obtained for both symmetrically and unsymmetrically laminated plates.

Mathematical Formulation

Recently, a number of studies have been conducted on the sensitivity of various static and dynamic aeroelastic responses (e.g., flutter, divergence, etc.) to four wing shape variables such as surface area, taper ratio, aspect ratio and sweep angle (e.g., Kapania et al.[1,2]). All these studies required the derivatives of the stiffness and mass matrices with respect to the four shape design variables mentioned previously. These derivatives were obtained using a finite-difference approach. Such an approach, though very easy to implement, suffers from one major drawback: the results may be extremely sensitive to the step size. A larger step size leads to significant truncation errors and a too-small step size may lead to round-off errors. To avoid these problems, it is desired that the derivatives be obtained analytically as far as possible.

In addition to the sensitivity of the stiffness and mass matrices, the sensitivity of the modal response (free vibrations and mode shapes) is also of interest. Sheena and Karpel[3] performed the static aeroelastic analysis of wings using free vibration modes. Recently, Karpel[4] also obtained the sensitivity derivatives of flutter characteristics and stability margins for aeroservoelastic design of wings by representing the wing in terms of its modal coordinates. The sensitivity analysis of aeroelastic responses therefore needs the sensitivity of the modal response. Accurate and efficient determination of this sensitivity information for the case of a generally laminated wing is the key objective of this research.

It is highly desirable in optimization to be able to calculate the effect of design variable changes without having to perform a full analysis for each design iteration. This need has

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led to an increased interest in accurate and efficient approximation techniques. Many optimization procedures use first-order Taylor series approximations of the objective function and constraints for this purpose. Pritchard and Adelman[5] recognized that the formulas for the sensitivity derivatives of system behavior variables can be interpreted as differential equations that may be solved to obtain closed form exponential approximations. On this basis, they developed the differential equation-based method, which they demonstrated for frequency and mode shape approximations.

Methods for first-order design sensitivities have been developed by Fox and Kapoor,[6] who presented exact expressions for the rates of change of eigenvalues and eigenvectors with respect to the design parameters of the actual structure, and indicated that the derivatives could be used successfully to approximate the analysis of new designs.

The present study presents first-order shape sensitivities of the natural frequencies and mode shapes of generally laminated tapered skew plates. The vibration frequencies and mode shapes of the plates are computed using the Rayleigh-Ritz method (Kapania and Singhvi[7]). Appropriate springs with large stiffness coefficients are used to simulate the effect of the geometric boundary conditions. Displacement functions are taken in the form of Chebyshev polynomials in combination with natural coordinates. This method permits one to perform sensitivity analysis of the vibration frequencies and mode shapes with respect to the shape parameters like the aspect ratio, sweep angle, taper ratio and surface area. To the best of the authors' knowledge, no such study has been performed for generally laminated skew plates. Once their first-order derivatives are obtained, the eigenvalues and eigenvectors are approximated using the technique developed by Pritchard and Adelman.[5] The approximate results are compared with those obtained by actual reanalysis and found to be in good agreement.

Sensitivities of Eigenvalues and Eigenvectors

The self-adjoint eigenvalue problem for undamped systems in structural dynamics is given by $[K]\{\phi_i\} = \lambda_i[M]\{\phi_i\}$, where the scalar quantities λ_i (eigenvalues) and the corresponding nontrivial vectors $\{\phi_i\}$ are to be determined. In the common structural application, the $n \times n$ matrices $[K]$ and $[M]$ are, respectively, the stiffness and mass matrices. Their order n corresponds to the number of elastic degrees of freedom (DOF) of the system. The sensitivity of the ith eigenvalue λ_i and the ith eigenvector $\{\phi_i\}$ with respect to various shape parameters (area, aspect ratio, taper ratio, and sweep angle) is determined both analytically as well as by the finite-difference (central-difference) method. The analytical formulation is explained in detail in this section.

As given by Fox and Kapoor,[6] the eigenvalue derivative is

$$\frac{\partial\lambda_i}{\partial\nu} = \{\phi_i\}^T\left(\frac{\partial[K]}{\partial\nu} - \frac{\partial[M]}{\partial\nu}\lambda_i\right)\{\phi_i\} \tag{1}$$

Here $\{\phi_i\}$, the ith eigenvector, is normalized with respect to the mass matrix such that $\{\phi_i\}^T[M]\{\phi_i\} = 1$. Note that the expression of Eq. (1) involves only the eigenvalue and eigenvector under consideration, and thus a complete solution of the eigenproblem is not needed to obtain these derivatives. The eigenvector derivative[6] is

$$\frac{\partial\{\phi_i\}}{\partial\nu} = \sum_{j=1}^{n} \alpha_{ij}\{\phi_j\} \tag{2}$$

where

$$\alpha_{ij} = \frac{\{\phi_j\}^T\left(\frac{\partial[K]}{\partial\nu} - \lambda_i\frac{\partial[M]}{\partial\nu}\right)\{\phi_i\}}{(\lambda_i - \lambda_j)} \qquad j \neq i \tag{3}$$

$$\alpha_{ii} = -\frac{1}{2}\left(\{\phi_i\}^T\frac{\partial[M]}{\partial\nu}\{\phi_i\}\right) \tag{4}$$

The details of the derivation of the stiffness and mass matrices are given in Ref. 7 and those of their derivatives are given in Ref. 8.

Approximation of Frequencies and Mode Shapes

As given by Pritchard and Adelman,[5] the equation for the derivative of the ith vibration eigenvalue λ_i with respect to a design variable ν can also be written as

$$\frac{d\lambda_i}{d\nu} = b - a\lambda_i \tag{5}$$

where values of a and b are given by Eq. (1). Equation (5) may be interpreted as a first-order differential equation in λ_i with variable coefficients. However, if a and b do not vary extensively with ν, then they may be evaluated at the nominal design and considered constant. After applying the nominal condition that $\lambda = \lambda_0$ when $\nu = \nu_0$, the general solution to Eq. (5), provided a is nonzero, is

$$\lambda_i = (\lambda_{i0} - b/a)\exp[-a(\nu - \nu_0)] + b/a \tag{6}$$

For the case where $a = 0$, the method produces the linear Taylor series approximation. The exponential term can be approximated by expanding it up to three terms and neglecting higher order terms.[8] Later, it is seen that this approximation is sufficiently accurate. The equation for the derivative of the mode shape $\{\phi_i\}$, with respect to a design variable can also be written as[5]

$$\frac{d\{\phi_i\}}{d\nu} = \{Q\} + D\{\phi_i\} \tag{7}$$

where the vector $\{Q\}$ and D can be determined by comparing Eq. (7) with Eq. (2). Equation (7) is a nonlinear first-order vector differential equation with variable coefficients. In order to solve this equation, D and $\{Q\}$ are evaluated at the nominal design and held constant. After applying the nominal condition that $\{\phi_i\} = \{\phi_i\}_0$ when $\nu = \nu_0$, we get

$$\{\phi_i\} = \left(\{\phi_i\}_0 + \frac{1}{D}\{Q\}\right)\exp[D(\nu - \nu_0)] - \frac{1}{D}\{Q\} \tag{8}$$

Equation (8) is a vector equation; it is uncoupled in the sense that each component of $\{\phi_i\}$ varies independently with the design variable ν. The difference between components is reflected in the corresponding components of $\{\phi_i\}_0$ and $\{Q\}$.

Results and Discussion

Derivatives of the mass and stiffness matrices, eigenvalues, and eigenvectors are computed using three different step-sizes in the central-difference method. These are compared with those obtained analytically. Approximations to the frequencies and mode shapes are implemented and tested. Both symmetric [0/25/25/0] as well as unsymmetric [0/25] laminate configurations are treated. The test cases involve perturbations of the surface area, aspect ratio, taper ratio, and sweep angle. A laminated cantilevered plate is the subject of our study. The thickness of each lamina is 0.0246 in. (0.00062484 m). The nominal surface area of the plate is 64 in.^{2} (0.0412902 m^2) with a nominal taper ratio (root chord/tip chord) of 0.75. The nominal aspect ratio, defined as (span^2/area) and not as (span/root chord), is taken to be 3. The plate is swept back at an angle of 15 deg at the quarter chord point. As per the standard convention, the fibers are assumed to be swept along with the plate. The material properties of the boron/epoxy used are $E_1 = 23.3 \times 10^6$ psi (161.21 GPa); $E_2 = 1.81 \times 10^6$ psi (12.52 GPa); $G_{12} = 0.976 \times 10^6$ psi (6.75 GPa); ν_{12}

Table 1 **Comparison of analytical and central difference shape sensitivities of the first four eigenvalues**

	Central difference			
Nominal value	0.01%	0.1%	1.0%	Analytical
		Area $[0/25]_s$		
0.36565e + 05	−0.11411e + 04	−0.11411e + 04	−0.11412e + 04	−0.11313e + 04
0.35835e + 06	−0.11172e + 05	−0.11175e + 05	−0.11176e + 05	−0.11174e + 05
0.12841e + 07	−0.40000e + 05	−0.40062e + 05	−0.40056e + 05	−0.40060e + 05
0.35792e + 07	−0.11156e + 06	−0.11159e + 06	−0.11159e + 06	−0.11162e + 06
		Taper ratio $[0/25]_s$		
0.36565e + 05	−0.31333e + 05	−0.31413e + 05	−0.31409e + 05	−0.31420e + 05
0.35835e + 06	−0.50267e + 06	−0.50293e + 06	−0.50297e + 06	−0.50287e + 06
0.12841e + 07	−0.34667e + 06	−0.34667e + 06	−0.34733e + 06	−0.34691e + 06
0.35792e + 07	−0.26133e + 07	−0.26147e + 07	−0.26149e + 07	−0.26141e + 07
		Aspect ratio [0/25]		
0.51717e + 04	−0.37300e + 04	−0.37293e + 04	−0.37295e + 04	−0.37216e + 04
0.79016e + 05	−0.14600e + 05	−0.14600e + 05	−0.14598e + 05	−0.14604e + 05
0.20628e + 06	−0.13467e + 06	−0.13470e + 06	−0.13471e + 06	−0.13457e + 06
0.67505e + 06	−0.19700e + 06	−0.19717e + 06	−0.19717e + 06	−0.19699e + 06
		Sweep [0/25]		
0.51717e + 04	−0.18733e + 03	−0.18693e + 03	−0.18691e + 03	−0.18794e + 03
0.79016e + 05	−0.77333e + 03	−0.77333e + 03	−0.77367e + 03	−0.77352e + 03
0.20628e + 06	−0.28667e + 04	−0.28600e + 04	−0.28593e + 04	−0.28657e + 04
0.67505e + 06	−0.19067e + 05	−0.19080e + 05	−0.19082e + 05	−0.19084e + 05

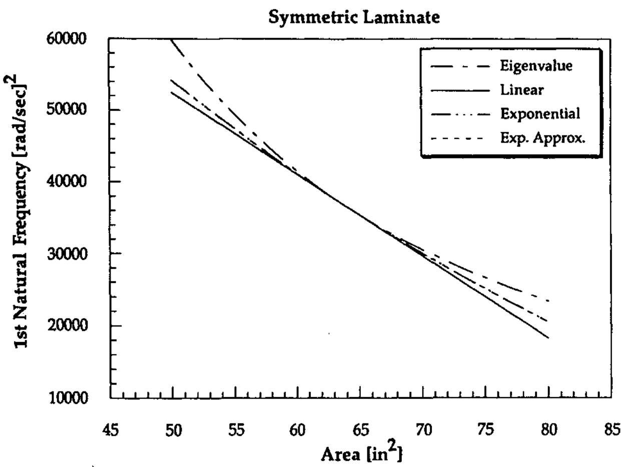

Fig. 1 Sensitivity of first mode of symmetric laminate with respect to area.

= 0.22; density = 0.069 lb/in.^{3} (1881.81 kg/m^3). Some key results for the symmetrically laminated plate are given here, and complete results for both the symmetrically and unsymmetrically laminated plates are given in Ref. 8.

Table 1 compares the analytical and finite-difference first-order sensitivities of the first four eigenvalues $\lambda = \omega^2$ where ω is the natural frequency in rad/s. In general, greater consistency is observed between values obtained for step-sizes of 0.1% and 1.0% than for one of 0.01%. The analytical derivatives are found to be in satisfactory agreement with the central-difference ones. To give an idea of the magnitude of the original eigenvalues computed at the base design, they are listed alongside their corresponding derivatives. To save space, the results for the sensitivities of the eigenvectors with respect to various parameters are given in Ref. 8.

The eigenvalues and eigenvectors are approximated using three different methods and compared with the true values obtained by reanalysis. Once the slope is determined, a linear approximation to the frequencies can be immediately plotted. Using the method developed by Pritchard and Adelman,[5] an exponential approximation can be obtained once the values of a and b in Eq. (6) are computed. And finally, a pseudoexponential scheme is adopted wherein the exponential is expanded up to three terms.[8]

In Fig. 1, the eigenvalues are plotted as a function of plate area for a symmetrically laminated plate. It is found that the exponential and the three-term expansion of the exponential yield either identical or almost identical curves. The linear approximation, as expected, is not as accurate as the exponential approximations. As can be seen from Fig. 1, the first eigenvalue decrease smoothly with an increase in the plate area for a symmetrically laminated plate. Results for second mode are given in Ref. 8. Also, the results for the approximation of the natural frequencies with respect to the other three parameters, along with the results for the approximation of a dominant coefficient in the first eigenvector with respect to all four parameters, are given in Ref. 8.

Concluding Remarks

The first-order shape sensitivities of the free vibration response of symmetrically and unsymmetrically laminated tapered skew plates have been studied. Shape derivatives of the stiffness and mass matrices, eigenvalues, and eigenvectors have been calculated using the central-difference method, with various step-sizes, as well as analytically. Approximations to the eigenvalues and eigenvectors have been obtained using three different methods and compared with the corresponding true values obtained by reanalysis. The eigenvalues demonstrated similar decreasing trends for changes in the area, aspect ratio, and taper ratio, whereas, the sweep angle variation resulted in a different decreasing trend. The dominant coefficient of the fundamental eigenvector decreased with an increase in the plate area, aspect ratio, and sweep, whereas it increased with a taper ratio increase. In general, the exponential schemes yielded good approximations, whereas the linear method was not as accurate.

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Multiple Pole Rational-Function Approximations for Unsteady Aerodynamics

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Nomenclature

$[A]$ = coefficient matrices
b = reference length
b_n = poles (lag-parameters)
$[Q]$ = unsteady aerodynamic transfer-function matrix
Q_{ij} = element (i, j) of $[Q]$
s = Laplace variable
U = freestream velocity

Introduction

FOR a general aeroservoelastic analysis, the equations of motion are desired in a linear, time-invariant, state-space form. This necessitates the representation of the unsteady aerodynamic transfer function matrix, for a general motion in the Laplace domain, by a rational-function approximation (RFA) for each term of the matrix. Since the unsteady aerodynamic transfer-function matrix $[Q(s)]$ is analytic for a causal, stable, and linear system, it can be directly deduced from the frequency domain data through a process of analytic continuation, which involves a least squares curve-fit.

Several approaches have been used to determine the poles (lag-parameters) of $[Q(s)]$ by a nonlinear optimization process. Dunn,[1] Karpel,[2] and Peterson and Crawley,[3] used gradient-based optimization schemes, whereas, Refs. 4–6, and 9 employed Simplex nongradient techniques. Peterson and Crawley[3] observed the phenomenon of repeated poles in approximating for the Theodorsen function. However, the repeated lag-states mistakenly indicated that the same fit-accuracy can be achieved by reducing the number of lag-states. Eversman and Tewari[5] encountered the repeated values of lag-parameters frequently in a nongradient optimized RFA, and correctly identified the phenomenon to indicate the need for a new multiple-pole approximation in the Laplace domain. Reference 5 showed that while the conventional approximation of simple poles produces an ill-conditioned eigenvalue problem for the state-space model when the poles are close to one another, the new multiple-pole RFA accounts for such cases consistently. Additionally, the use of multiple-poles resulted in a large reduction in the optimization cost, while preserving the fit-accuracy and the total number of aerodynamic states when compared to the conventional approximation. Eversman and Tewari[6] also presented improved and consistent RFA for the Theodorsen function by using the multiple-pole approximation. Tewari,[9] in a Ph.D. dissertation, showed that the multiple-pole RFA is needed not only in the subsonic regime, but also for supersonic speeds. References 5 and 9 arrived at the multiple-pole RFA through numerical considerations. The present work examines the multiple-pole RFA from a mathematical standpoint and validates its need by concluding that multiple-pole RFA is dictated in the function space by the constrained optimization theory.

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Numerical Need for Multiple-Pole RFA

A simple-pole, least-squares RFA for the unsteady aerodynamic transfer-function matrix can be expressed as

$$[Q(s)] = [A_0] + [A_1]s(b/U) + [A_2]s^2(b/U)^2 + (U/b)\sum_{n=1}^{nL}\frac{[A_{(n+2)}]}{s + (b/U)b_n} \tag{1}$$

Reference 5 showed that the optimized values of two or more lag parameters b_n frequently tend to be close to one another for a subsonic numerical test case. It was also shown that when repeated poles occur, numerical considerations point toward the need for a multiple-pole RFA, given by

$$[Q(s)] = [A_0] + [A_1]s(b/U) + [A_2]s^2(b/U)^2 + (U/b) \cdot\sum_{n=1}^{N_1}\frac{[A_{(n+2)}]}{s + (U/b)b_n} + (U/b)^2\sum_{n=N_1+1}^{N_2}\frac{[A_{(n+2)}]}{[s + (U/b)b_n]^2} + \dots \tag{2}$$

where N_1 is the total number of poles, $(N_2 - N_1)$ the number of poles repeated twice or more times, etc. Such RFA avoid the ill-conditioned eigenvalue problem produced by having repeated poles in Eq. (1).

While Ref. 5 studied the RFA for the subsonic case, it was subsequently discovered[9] that repeated poles are equally prevalent in the supersonic regime, and that a multiple-pole RFA, given by Eq. (2), produces a consistent and efficient approximation for supersonic speeds. In the test-case considered, the same planform geometry was used as in Ref. 5, but the wing was stiffened in order to have the flutter-speed in the supersonic regime. The supersonic "doublet-point" method[7–9] was used to generate the frequency domain data at the following set of reduced frequencies:

0.0, 0.05, 0.1, 0.125, 0.175

0.2, 0.3, 0.5, 0.7, 0.9, 1.2

Only the first six structural modes were retained. Table 1 presents the optimum pole values for supersonic Mach numbers. As in Ref. 5 for subsonic Mach numbers, it is noted